DOCUMENT RESUME

ED 120 026 SE 020 626

AUTHOR Junge, Charlotte W.; And Others
TITLE Mathematics for Young Children. A Summary of Research and Related Literature.
INSTITUTION Florida State Dept. of Education, Tallahassee. Early Childhood and Elementary Education Section.
PUB DATE 75
NOTE 33p.; Prepared for a national leadership conference on mathematics for early childhood

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage
DESCRIPTORS *Annotated Bibliographies; Bibliographies; *Cognitive Development; Curriculum; *Early Childhood Education; Elementary Education; *Elementary School Mathematics; Evaluation; Instruction; Learning; Mathematics Education; Primary Education; *Research Reviews (Publications)

ABSTRACT

This booklet provides an annotated list of 48 research reports related to mathematics teaching and learning of primary school children, and a bibliographic listing of 102 additional papers dealing with these topics. Each bibliography is organized into five major divisions: (1) growth and development of children, (2) development of mathematical thought in children, (3) mathematical content appropriate for young children, (4) strategies of teaching mathematical concepts to young children, and (5) assessment and evaluation of mathematical learning. (SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U S DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ED120026

MATHEMATICS

FOR

YOUNG

CHILDREN

This public document was promulgated at an annual cost of $297.67 or $.29 per copy to provide educators with selected, condensed research summaries regarding mathematics and the young child.

SE 020 626

MATHEMATICS

FOR

YOUNG CHILDREN

A Summary of Research
and Related Literature

1975

Prepared by

Dr. Charlotte W. Junge, Professor Emeritus, Wayne State University, Wayne, Nebraska

Patricia Campbell, Graduate Student, Florida State University, Tallahassee, Florida

Dr Thomas Denmark, Associate Director, Project for the Mathematical Development of Children, Florida State University, Tallahassee, Florida

for

A national leadership
conference on mathematics
for early childhood

Cosponsored by the
NATIONAL COUNCIL
OF TEACHERS OF MATHEMATICS
and the
ASSOCIATION FOR CHILDHOOD
EDUCATION INTERNATIONAL

We would like to acknowledge Ms. Patricia Campbell, Dr. Charlotte Junge and Dr. Thomas Denmark for granting us permission to reproduce this document for use by the Florida Department of Education.

For further information contact:

Florida Department of Education
Early Childhood and Elementary
Education Section
Knott Building
Tallahassee, Florida 32304

MATHEMATICS EDUCATION FOR YOUNG CHILDREN*

RESEARCH FOUNDATIONS

PART I: PROLOGUE

The development of sound mathematical programs for young children requires careful study and research. The identification of objectives, which take into account the importance and relevance of mathematical learning for the yound child -- and for society -- is a difficult and complex process. For example:

1. Certain objectives relate to the nature of mathematics, itself. Mathematics is a science, a social science, a language, and a way of thinking about space, form and quantity;

2. Certain objectives are cognitive and relate to basic mental processes involved in learning. These are concerned with the role of "play" and intuition, with abstracting, classifying, and generalizing ideas, and with the use of symbolic thought in mathematical learning;

3. Other objectives are individual and personal and relate to research findings with reference to the physical, social, intellectual and affective growth and development of the child;

4. Still other objectives are pedagogical in nature and have clear reference for the selection and organization of mathematical learnings into continuous learning sequences, and for the teaching strategies employed in successfully presenting these learnings to the child.

---

*Prepared by Charlotte W. Junge, with the assistance and cooperation of Patricia Campbell and Thomas Denmark (mathematics Project, Florida State University).

The question of the relative emphasis to be given these different objectives in planning and conducting satisfactory mathematical programs for young children need not be left to chance, nor to the individual preferences of teachers and curriculum makers. Current literature in child growth and development and mathematics education can provide broad guidelines for curriculum and instruction.

The following, selected, condensed research summaries are presented in the hope they will provide conference participants with a background of the current thinking regarding mathematics and the young child, and that this background will be helpful in conference discussions. The literature reviewed is representative of the great amount of data available to the serious educator, and is in no way all-inclusive.

PART II: RESEARCH SUMMARIES

AREA 1: Studies Related to the Growth and Development of Children

Current literature indicates the crucial importance of the experiences of the infant and young child in the individual's capacity to function in later life. Emergent programs in Early Childhood Education, having language development as their core, incorporate basic concern for affective and cognitive development.

Braun, Frederick G. The child from six to eight -- developmental growth patterns. Paper presented at the meeting of the International Reading Association, Anaheim, California, May 6-9, 1970. (ERIC Document Reproduction Service No ED 045 325.)

The author presents his assessment as to why children have reading problems and makes suggestions for improving reading instruction. Specifically, the author states that current practices in many classrooms interfere with growth patterns on which the ability to read is dependent. To alleviate this the teacher should have more freedom to interact with individual children, greater flexibility in scheduling and adaptation of materials to fit the child, and further training to become a better decision-maker in the areas of curriculum, instruction, and management. The classroom environment must be arranged to provide for self-directedness through child planning and decision-making to encourage the building of a positive self-concept by creating an atmosphere which is free of fear and failure. References are included.

Cordis, Leora L. Learning abilities of the prekindergarten child. Final report. Eugene, Oregon: University of Oregon, 1972. (ERIC Document Reproduction Service No. ED 066 225.)

Report of a study which collected data from which the cognitive processes of the prekindergarten child could be analyzed. 53 children, aged $3\frac{1}{2}$ to $4\frac{1}{2}$ years, interacted with the Kindergarten Evaluation of Learning Potential (KELP) instrument for approximately twenty

minutes each day over a twelve week period. At age 4½, children are significantly better able to conceptualize preacademic skills than at age 4. This supports Piaget's theory that children develop the ability to generalize at about age 4.

Elkind, David. Misunderstanding about how children learn. Today's Education, 61, 1972, 18-20.

The author describes five common misconceptions about how children learn and discusses the implications for curriculum development. The following is a brief summary of these misconceptions and Elkind's view of learning. (1) Young children are most like adults in their thinking and least like adults in their feelings. Since children are often quite capable with linguistic skills, adults overestimate their capacity to think. (2) Children learn best while sitting still and listening. Actually a child learns through engaging in real actions involving tangible objects. (3) A child can learn and operate according to rules. This is not the case. Young children of primary age cannot learn rules. (4) Acceleration is preferable to elaboration. A child who elaborates the skills he possesses is more likely to be better prepared for future learning than a child who learns a great deal in a short time and does not assimilate what he has learned. (5) Parents and teachers can raise a child's IQ. Enrichment does not increase intellectual ability, but it does affect how a child uses his ability.

Heiser, R. B. Some notes on concept formation in young children. Journal of Genetic Psychology, 114, 1969, 219-27.

In order to investigate the question "How do children think about language in its spoken and written forms?" twelve 5-6 year olds were interviewed. These children, entering the first grade, did not understand the meaning of such basic terminology as "word" and "sound." Testing at the end of the year revealed, that the children had made little progress towards developing a meaningful understnading of these concepts. The children also confused the meaning of such word pairs as "letters and numbers" or "write and draw."

Klein, Robert A. Cognitive growth in young children: Some theoretical implications pertaining to identity, language and memory. Paper presented at the meeting of the UAP-USC Conference on Piagetian Theory and the Helping Professions, Los Angeles, February 16, 1975. (ERIC Document Reproduction Service No. ED 076 238.)

Language as an identifiable cognitive behavior must be studied in relation to identity and memory, all of those structures undergo progressive changes as the child develops. The organization of the development of the organism depends upon relatively orderly structures of growth, following foreseeable pathways or creodes. The process occurring within each creode, however, are susceptible to certain environmental modifications. The acquisition of language parallels

the development of identity and renders it meaningful. The style of reasoning used by a child is very much related to the language used and a close relationship exists between the structure of a term and the developmental stages of seriation. Cognitive operations never exist in isolation; the acquisition of one enhances or potentiates the acquisition of another.

The development of the memory schema lies within the development confines of identity and language. It is only under the circumstances of the changed and changing schema that memory becomes not more accurate but more in concert with the other same-level cognitions and modalities of thinking. The observation of qualitative differences of behavior at differen chronological levels establishes memory as possessing the structure(s) through which identity is assimilated and language accommodated. Memory, and its manifestation via increasingly accurate reproduction of the original stimulus, is an integrative and integrated factor in cognitive growth.

Lichtenberg, Philip, and Norton, Dolores G. Cognitive and mental development in the first five years of life: A review of recent research. Washington: U. S. Government Printing Office, 1970. (ERIC Document Reproduction Service No. 044 754.)

This is a survey of studies sponsored by National Institute of Mental Health. The authors identified and correlated ideas, themes, perspectives and issues related to child development. Growth and development happens when a child is engaged with actions with other people. Growth and development was most effective when conditions are playful and pleasurable, as opposed to imposed training in the proper way. Disadvantaged children are turned off by failure to create such actions. Action is not just motion; it has certain specific characteristics: (1) Action is double directed, that is, each child relates to himself and to others at the same time. (2) Action binds the child to people and to things. (3) Action involves mutuality and reciprocity between the child and his caretaker. Research seems to reject development as a simple maturational process, since action is a reciprocal relation; further, recent research does not support the notion that a child is infinitely mallable, since action must be mutual.

The stimulation for action must be within the range of a child's capabilities. However, stimuli with high uncertainty of outcome are more effective. Satisfaction and fulfillment are generators of development. The thrusts for action must be open for a child to explore and directed to move him forward.

Implications for cognition development are:

(1) There are inherent characteristics which affect cognitive development.
(2) Cognitive development involves a maturational process in hierarchial stages, each stage building on previous stages.

(3) Cognition growth occurs through motivation to master the new and the more difficult.
(4) Cognitive development is highly influenced by a child's interaction with his social and physical environments.
(5) A child moves from dominances by his environment to dominance by his own capabilities through cognitive development.
(6) Cognition grows as a child becomes more differentiated.
(7) Language development is related to cognitive development.

Moore, Ruth C., and Ogletree, Earl J. A comparison of the readiness and intelligence of first grade children with and without a full year of Head Start training. Education, 93, 1973, 266-270.

The purpose of this study was to investigate the development of readiness and IQ performance among Head Start pupils as compared with non-Head Start pupils. The subjects were 192 first graders from six inner city Chicago schools. All of the children came from the same community and were considered to be socially disadvantaged. The results of the study indicated that readiness, as measured by the Metropolitan Readiness Test, and intelligance test performance, as measured by the Kuhlman-Anderson Intelligence Test, was significantly higher among children who had a full year of Head Start. And that these gains were stabilized during the first year of school.

Gordon, Wells. Learning to code experience through language: An approach to the study of language acquisition. Revision of a paper presented at the meeting of the London School of Economics, February, 1973. (ERIC Document Reproduction Service No. ED 081 286.)

"How does a child learn to relate to his own experiences to the formal means of communication?" To answer this question, the author conducted taped interviews with seven children. Based on the subsequent analysis of the tapes, the author maintained that the innate pre-dispositions that underlie the development of cognitive ability to organize and structure experience, also underlie language development.

Witz, E. G., and Easley, J. A., Jr. Analysis of cognitive behavior in children. Final report. University of Illinois Curriculum Laboratory. Urbana: University of Illinois Press, 1972. (ERIC Document Reproduction Service No. ED 076 261.)

Final report of a project which attempted to conceptualize and map cognitive processes and structure. Their conception of cognitive structure and of the educational process and their perspective on cognitive functioning differs fundamentally from Piaget and other investigators. In this study, cognitive structures are viewed as entities, which are usually presented by a mathematical object. Such cognitive structures exist in a child for some period of time; they may or may not be activated. If activated, the cognitive structure can be assimilated in a given situation and evidence of this activation would be observable in the child's behavior. A

variety of cognitive structure types exist, each with its own characteristics. These types of cognitive structures exist within each individual at a certain period of development. The cognitive structures may be discrete or continuous. Growth of knowledge is the integration of cognitive structures of each type into a larger structure.

AREA 2: Studies Related to the Development of Mathematical Thought in Children

It is believed that human learning passes through certain sequential stages which parallel the growth of ability in children to deal with mathematical ideas in a logical fashion. The rate at which a learner moves through these stages varies with the individual learner.

Baker, Nancy E., and Sullivan, Edmund V. The influence of some task variables and of socioeconomic class on the manifestation of conservation of number. Journal of Genetic Psychology, 114, 1970, 21-30.

A total of 156 kindergarten children were given Piagetian number conservation tasks involving conservation of inequality. Partial confirmation was given to the hypotheses that conservation is more likely (a) with high interest task materials (candy, toys) than with low interest materials (beans, checkers), and (b) with smaller aggregate sizes. Conservation ability was manifested significantly more often by middle-class than by lower-class children. A positive correlation was found between conservation performance on a task involving addition and subtraction concepts.

Carpenter, Thomas Phelps. The role of equivalence and order relations in the development and coordination of the concepts of unit size and number of units in selected conservation type measurement problems. (Unpublished doctoral dissertation, University of Wisconsin, 1971.) Dissertation Abstracts International, 32, 1972, 448A.

This study was designed to investigate the development of certain measurement concepts, to relate this to the development of conservation, and to determine the role of equivalence and nonequivalence relations in conservation and measurement problems. 218 subjects in grades K-2 were individually tested on four to nine items selected from a set of eighteen conservation and measurement problems. Each problem was administered in situations employing different combinations of equivalence or nonequivalence relations. Results were: (1) No significant difference in the problems due to combinations of equivalence or nonequivalence relations; (2) Most first and second graders could use measurement to compare quantities, however, many do not understand the need for a constant unit of measure and err when using more than one unit of measure; (3) Numerical conflicts, due to different units of measure, resulted in the same degree of error as visual conflict, due to pouring liquids into

different shaped containers; and (4) Most errors result from the child entering on an immediate dominant dimension.

Friedman, Myles, et. al. The development of problem-solving skills in early childhood. Committee on Educational Research. Columbia: University of South Carolina, 1971. (ERIC Document Reproduction Service No. ED 057 073.)

This investigation was designed to identify scales indicative of the development of problem-solving behavior in young children and to discover whether children of different backgrounds exhibit similarities in the order of development and levels of achievement of problem-solving behaviors. 1,430 children (770 from "advantaged" economic backgrounds and 660 from "disadvantaged" economic backgrounds) ranging from four to six years of age (same number of children in each three month interval in the age range), were tested on items from twenty-two listed. Conclusions were: (1) same order among children of extremely different backgrounds; (2) There are particular problem-solving and disadvantaged children; and (3) Many items on the tests did not scale reliable for the disadvantaged children.

Haney, Joanne H., and Hooper, Frank H. A developmental comparison of social class and verbal ability influences on Piagetian tasks. Journal of Genetic Psychology, 122, 1973, 235-245.

This study attempted to evaluate the influences of social class and verbal ability as determinants of performance on Piagetian concrete operations tasks. A sample of 160 subjects (kindergarten to fourth grade) were matched on socioeconomic status and verbal ability levels. A battery of Piagetian tasks was administered. Results indicated: an absence of significant socioeconomic status effects for any of the Piagetian tasks; a significant main effect for the verbal ability and age grade level factors on the majority of tasks excluding unidimensional height seriation; and a general absence of sex main effects on higher order interactions. The lack of differences with regard to socioeconomic status levels indicates that previous research which revealed such influences may have had socioeconomic status confounded to varying degrees with differences in underlying verbal skills.

Koshinsky, Claudia, and Hall, Alfred E. The developmental relationship between identity and equivalence conservation. Journal of Experimental Child Psychology, 15, 1973, 419-424.

Investigated the previous finding that identity conservation develops prior to equivalence conservation. Seventy-two kindergarten, first and second graders were given three conservation tasks: (1) Identity; (2) Equivalence 1, in which the perceptual cues were comparable to those in Identity; and (3) Equivalence 2, the traditional Piagetian conservation task. Each task was administered under two levels of transformation. 86% conserved in an all-or-none fashion, and 10%

passed equivalence while failing identity, thus contradicting the developmental priority of identity conservation.

Little, Audrey. A longitudinal study of cognitive development in young children. Child Development, 43, 1972, 1024-1036.

The purpose of the study was to investigate the relationship between intelligence and performance in Piagetian-type tasks, to explore the relative effects of CA and MA on performance of cognitive tasks, to determine whether acquisition of conservation, seriation, and class inclusion occurred concurrently or in an invariant order, and to examine the response patterns for characteristics which differentiated between IQ groups. Eighty children of "average" and "superior" IQ were given a series of 3 conservation, 3 seriation, and 3 class inclusion tasks at age $4\frac{1}{2}$-5 years and again when aged $6\frac{1}{2}$-7 years. Comparisons revealed many qualitative differences at $4\frac{1}{2}$-5 years level, both qualitative and "stage" differences at $6\frac{1}{2}$-7 year level.

Overbeck, Carla, and Schwartz, Marian. Training in conservation of weight. Journal of Experimental Child Psychology, 9, 1970, 253-264.

Forty-eight 5-8 year old nonconservers of weight were divided into four groups and given four training treatments for conservation of weight. The training variables were: presence vs. absence of reinforcement and active vs. passive participation. Reinforcement facilitated acquisition but active participation had no effect. It was concluded that conservation of weight was learned through the use of reinforcement in the training procedures used.

Papalia, Diane E., and Hooper, Frank H. A developmental comparison of identity and equivalence conservations. Journal of Experimental Child Psychology, 12, 1971, 347-361.

Investigated the developmental priority of identity conservation as contrasted with equivalence conservation using quantity and number conservation tasks. Subjects were 60 four, five, and six-year old children of middle socioeconomic class background. It was concluded that identity concepts develop prior to equivalence concepts when the content area is quantity conservation. In contrast, clear-cut conclusions cannot be made about number conservation.

Schnall, Melvyn, Atler, Ellen, Swanlund, Terry, and Schweitzer, Thomas. A sensory-motor context affecting performance in a conservation task: A closer analogue of reversibility than empirical return. Child Development, 43, 1972, 1012-1023.

In a study involving 80 5-7 year old nonconservers of quantity, Schnall et. al. investigated Piaget's distinction between "empirical return" and reversibility in relation to conservation. The subjects underwent three trials on a conservation of quantity task under one of the following conditions: (a) Experimenter spread 1 row of

elements by hand; (b) Subject spread elements by hand; (c) Subject spread elements by stretching an elastic strip on which elements were attached; (d) Subject spread by means of a nonelastic cardboard device. The elasticity condition involving bidirectional tension modeled the characteristic of operational reversibility, and this condition alone led to significant induction of conservation judgments, thus supporting Piaget's distinction.

Van Wagenen, R. Keith, and Walker, Alice A. Formulation and quick integration of mathematics concepts in the child during the first school year. Tempe, Arizona; Arizona State University, 1973. (ERIC Document Reproduction Service No. ED 085 116.)

This document is the report of a study which used linear measurement as its basis for introductory mathematics. The subjects were taught the physical operations of linear measurement providing practice in addition, subtraction, and other concepts with little or no pencil and paper arithmetic. 161 children in the last half of kindergarten and in first grade received the measurement instruction. A comparison group from two comparable schools was taught under the usual curriculum. Throughout the study, treatment and comparison groups did not differ in general conservation performance. However, an ancillary study demonstrated that linear measurement competence is preliminary to linear conservation, contrary to Piaget's contention that a child must conserve before learning to measure. After five or six months of treatment, the measurement group was superior in measuring competence, but slightly inferior in "typical" first-grade mathematics.

Wang, Margaret C., et. al. The sequence of development of some early mathematics behaviors. Pittsburgh University: Learning Research and Development Center, 1970. (ERIC Document Reproduction Service No. ED 052 947.)

Sought to determine whether a number of specific counting and numeration behaviors emerge within children in a fixed developmental sequence; at what point in the development of mathematical behaviors the use of numerical representation normally appears; and what relationship holds between development of counting skills and development of one-to-one correspondence operations. The subjects were 78 kindergarten children in an urban public school. Results of analysis of performance on a battery of tests suggested: (1) A reliable sequence of skills in using numerals; (2) The dependence of learning numerals upon prior acquisition of counting skills for sets of the size represented; (3) Acquisition of numeral reading for small sets before learning to count larger sets; and (4) The indepencence of counting and one-to-one correspondence operations in young children.

Weaver, J. Fred. Some concerns abou the application of Piage 's theory and research to mathematical learning and instruction. Arithmetic Teacher, 19, 1972, 263-270.

The author expresses his concern over the "rapid growth of a cult that assigns to Piaget a messianic role in relation to education." Concerns are: (1) The need for inferences for instruction to differ from inferences based on Piaget's conclusions; (2) Common practice of educators to justify their programs by citing Piaget; and (3) Questions concerning Piaget's view of the relation between learning and development.

AREA 3: Studies of Mathematical Content Appropriate For Young Children

Current studies indicate key mathematical concepts having social and mathematical significance for young children. The studies emphasize the inter-locking nature of mathematical concepts and the importance of observing, discussing, and reasoning in the development of ideas fundamental to later learning.

Harvey, John G. The content of arithmetic included in a modern elementary mathematics project. Research and Development Center for Cognitive Learning. Madison: University of Wisconsin, 1971. (ERIC Document Reproduction Service No. ED 070 656.)

Details of arithmetic topics proposed for inclusion in a modern elementary mathematics program and a rationale for the selection of these topics are given. The sequence of topics is also discussed.

Heard, Ida May. Developing Geometric Concepts in the Kindergarten. The Arithmetic Teacher, 16, 1969, 229-230.

A study of twenty-five kindergarten students, from culturally advantaged homes, involved in a study of intuitive geometry. Children were pre-tested on the SMSG Fall Inventory Test. Content included (1) Matching, naming, and identifying the circle, triangle, square, and rectangle; (2) Differentiating the circle and circular region; (3) Ordering similar shapes according to size; (4) Constructing geometric shapes on 'nail boards' using rubber bands; and (5) Relating familiar geometric shapes to real life objects. Instruction was structured but informal. Active participation of the learner and concrete, manipulative aids characterized the instruction. Results indicate children learned and enjoyed this planned experience.

Hodgen, Laurel, et. al. School Before Six: A diagnostic approach. (2 vols.) St. Louis: The CEMREL Institute, 1974.

This text is actually a manual for nursery or kindergarten teachers who wish to incorporate the diagnostic approach into their philosophy and methods; it is a source of ideas presenting diagnostic procedures, assessment techniques, and instructional activities. Volume I describes the diagnostic philosophy of observation, assessment, and record keeping with an empsasis on procedures to assess the child's learning needs and strengths. Volume II is a collection of learning activities in the areas of motor skills, language development, and social-emotional development.

Lazarus, JoAnn Mannise. The development of a kindergarten mathematics program: Implemented through in-service teacher education. (Unpublished doctoral dissertation, Wayne State University, 1972.) Dissertation Abstracts International, 33; 1972, 220A-2221A.

The purpose of this study was to develop a kindergarten mathematics program implemented through in-service teacher's meetings. The author designed a program entitled One and One More which was used for one quarter by 277 kindergarten children, under eleven teachers, in ten Van Dyke, Michigan elementary schools. The teachers participated in three workshops; pre-tests and post-tests were administered. Results indicate that these kindergarten children benefited from the mathematics instruction. There was unequal achievement in the areas concerning weight, numeral sequence, memory and kinesthetic response, supporting the Piagetian concept of maturation and learning.

Pollio, Howard R., and Whitacre, Janet D. Some observations on the use of natural numbers by preschool children. Perceptual and Motor Skills, 30, 1970, 167-174.

The purpose of this study was to determine the pre-schooler's numerical skills, namely counting and numeration or conservation. Sixty children (35 children ages 3.5 to 5.4 enrolled in the University of Tennessee nursery school and 25 chileren aged 5.0 to 6.7 enrolled in the Knox County Head Start Project) were presented with a number of tasks designed to assess skills in counting and enumeration. Contrary to expectations, some counting tasks were more difficult than enumeration tasks, although both did show age trends. Results showed that children who could count from any given point (regardless of age) consistently performed better on all other numerical tasks, indicating that fluent counting may not depend primarily on rote factors. Findings seemed to support Piaget's analysis of numerical development.

Shah, Sair Ali. Selected geometric concepts taught to children ages seven to eleven. The Arithmetic Teacher, 16, 1969, 119.

Children ages seven to eleven were taught a series of lessons in geometry over a three week period. Teachers were given freedom to use whatever concrete materials they considered appropriate. Concepts taught included (1) Recognition of plane figures; (2) Nets; (3) Symmetry about a line; (4) Reflection; (5) Rotation;

(6) Translation; (7) Bending and stretching; and (8) Networks. Conclusions indicate performance by seven to eight age group was sometimes low; performance by eight to eleven age group was satisfactory and, in certain areas, high. Performances were better as the age range increased. Researcher believes the results indicate that this kind of geometry can be successfully studied by children of different ages.

Shah, Sair Ali. Selected topological concepts taught to children ages six to nine. A report read at the annual meet of the AERA, 1971.

In a follow-up of the prior study Shah found that first, second and third grade students could be successfully taught concepts of solids, sheets, lines, networks, order, and betweeness.

Steffe, Leslie., and Carey, Russell L. Equivalence and order relations as interrelated by four- and five-year old children. Journal of Research in Mathematics Education, 3, 1972, 77-88.

This study investigates interrelationships among four- and five-year old children's ability to (1) Conserve length relations not involving the assymmetric property and consequences; (2) Use the reflexive and non-reflexive properties of length relations at two different points in time. Conclusions show (a) that an ability to conserve length relations (not involving the assymetric property and consequences) precedes an ability to conserve length relations involving the assymetric property and consequences; (b) an ability to conserve length relations involving the assymetric property and consequences precedes an ability to use transitivity of length relations; and (c) an ability to use reflexive and non-reflexive properties is not related to an ability to conserve length relations.

Williford, Harold J. A study of transformational geometry instruction in the primary grades. Journal for Research in Mathematics Education, 3, 1972, 260-271.

The purpose of this study was to assess the performance of subjects who were given instruction in transformational geometry as compared with subjects who received minimal introductory instruction on transformational geometry terminology and concepts. 63 pupils randomly selected from a population of 106 average or above-average second and third graders from two schools in Jackson County, Georgia, were randomly assigned to the two treatment groups. The experimental group received twelve lessons and the control group received one lesson concerning rigid motion and congruence concepts. Results indicated that after instruction the experimental subjects performed significantly better on the geometry achievement tests, but no significant difference was noted on the spatial ability tests. This seems to imply that children can learn transformational geometry skills, but that they have difficulty applying these skills to more general tasks.

AREA 4: Studies Related to Strategies of Teaching Mathematical Concepts to Young Children

Research on teaching strategies explores the relative merits of expository, discovery, and programmed approaches to teaching and the impact of classroom organization on learning. Use of concrete and manipulative materials, laboratory and small group activity and individualized approaches to learning have been studied with reference to their value for teaching mathematics.

Anastasiow, Nicholas J., et. al. A comparison of guided discovery, discovery and didactic teaching of mathematics to kindergarten children. American Educational Research Journal 7, 1970, 493-509.

This study compared the guided discovery, discovery, and didactic (rule-example) teaching of mathematics. Forty-one kindergarten children (all but one black) from three classes in Durham, North Carolina, were taught the same basic math curriculum by one of the three teaching strategies. The teachers involved rotated, each eventually teaching all three strategies. Pre-tests, post-tests and retention tests were administered. Results indicate that the rule-example method (didactic) may be most efficient for mastery of content at the early stages of the curriculum, while guided discovery appears to be more efficient for mastery of more complex principles.

Engel, Carol L., and Lerch, Harold H. A comparison of first-grade children's abilities on two types of arithmetical practice exercises. School Science and Mathematics, 71, 1971, 327-334.

The purpose of this study was to determine whether first-grade children who had studied addition facts without emphasis on closed number sentences could make correct decisions about basic addition ideas stated as either true or false number sentences. 130 first graders from six classes in two schools in Urbana, Illinois were administered an author-made measurement instrument consisting of basic addition combinations (presented as both open number sentences and vertical problems) and closed number sentences which were to be labeled either true or false. Results indicate that these children could make decisions about the addition statements truth or falsity with a reasonable higher degree of accuracy.

Fennema, Elizabeth H. The relative effectiveness of a symbolic and concrete model in learning a mathematical principle. Journal For Research in Mathematics Education, 3, 1972, 233-238.

A study to determine whether there were differences in the learning of groups of children who learned a specified principle represented by a concrete or symbolic model. Ninety-five second-grade children, divided into eight groups were taught multiplication as union of equivalent, disjoint sets. At the end of the instructional period all groups of children had learned the principle to the point of direct recall. Groups of children who had learned with the symbolic model were able to transfer their learning better than those groups who had learned with the concrete model. Conclusions appear to indicate that for certain groups of children, learning a specific principle, a symbolic model may provide a better mode for enhancing learning than a concrete model.

Keane, Dorothy Louise. An exploration of the Nuffield Mathematics Teaching Project with reference to its current use in selected British schools. (Unpublished doctoral dissertation, Wayne State University, 1973). Dissertation Abstracts International, 34, 1973, 3029A.

This study describes the Nuffield Mathematics Teaching Project as it is currently in use in a sample of British schools. A sample of eleven schools, eight teachers' centers, eleven teachers, six headteachers and six wardens was chosen on the basis of the geographical location and socioeconomic status of the schools or centers. Data was collected by means of questionnaires, personal interviews and personal observations, along with examination of materials published by the Project and literature pertaining to the Project. Results depict the Nuffield Project as attempting to change the mathematics content and instructional strategies used with children aged five to thirteen. Emphasis was placed on a concrete approach to mathematics, discovery learning, and a need to relate mathematics to the experiences of the children. The major force behind the Project seems to be the teacher's centers which provide in-service education for teachers, as well as contributing to curriculum development.

Peters, Donald L. Discovery learning in kindergarten mathematics. Journal for Research in Mathematics Education, 1, 1970, 76-87.

Non-cued Discovery (NCD), Perceptual Cue Guided Discovery (PCG) and Verbal Didactic (VDI) instructional procedures

for teaching number conservation were compared with each other and with a control to evaluate the discovery learning hypothesis of greater and more enduring learning with ready transfer to new situations. One hundred and twenty kindergarten subjects from schools primarily populated by students from low socio-economic areas were randomly assigned to one of the four treatments after a pre-test on number conservation and transfer. The three experimental groups each received training in two sessions, the control group received no training. Post-tests indicated that the VDI group was superior to either discovery treatment. Both the PCG and the VDI treatments were superior to the NCD and the control groups on the retention measure. Although some transfer value of the training was apparent, no significant treatment differences emerged.

Romberg, Thomas A., and Gilbert, Lynn E. The effect of training on length of performance of kindergarten children on nonstandard related tasks. Journal of Research in Mathematics Education, 3, 1972, 69-76.

An exploratory study designed to examine whether explanation of terms via a process-by-attribute instructional approach would help children's performance on non-standard comparison tasks involving that attribute. A total of seventy-one children were in the study. The study was conducted with kindergarten children in a pre-test, post-test, non-equivalent control group design with another treatment group not receiving the pre-test. The treatment involved three days of brief instruction concerning the definition, comparison, and representation of length. Results indicate (a) that the process-by-attribute instruction in this study did yield significant performance gains; and (b) that it is difficult to force, or convince, or persuade, or teach children to abandon perceptual misconceptions which are a phenomenon of children's thinking and one not unique to particular subjects.

Stafford, Don G., and Renner, John W. SCIS helps the first grader to use logic in problem solving. School Science and Mathematics, 71, 1971, 159-164.

This study compared the effect of the Science Curriculum Improvement Study (SCIS) first grade program on the first grader's ability to use simple logic as evidenced by their performance on Piagetian conservation tasks. One hundred and twenty beginning first graders were assigned to two treatment groups utilizing either the SCIS program or a science program from a textbook series. The remainder of the curriculum was identical for the two groups. A Piagetian-type pretest involving six conservation tasks was administered during the first week of school; a post-test consisting of the same facts was given in the first week of the following February. Results indicate that the SCIS group scored significantly higher in every conservation task except area.

Weber, Audry Wheatly. Introducing mathematics to first grade children: Manipulative vs. paper and pencil. (Unpublished doctoral dissertation, University of California, Berkeley, 1969). Dissertation Abstracts International, 30, 1970, 3372A-3373A.

This study attempted to determine what effects two introductory methods of instruction and socioeconomic status have on the mathematical understanding of first graders. Two schools, designated as low socioeconomic status, and two schools, designated as middle socioeconomic status, were randomly selected; three classes from each socioeconomic group were randomly assigned to one of two treatment groups for thirty consecutive school days. The treatment groups received reinforcement of mathematical concepts by follow-up activities involving either pencil and paper or manipulative materials. Results from the pre- and post-test, Metropolitan Readiness Test, Forms A and B, indicate no significant difference between the methods. Results from an author made Oral Test of Understanding indicate that children from the manipulative materials group scored significantly higher, both in the number of correct responses and in the levels of understanding.

AREA 5: Studies Related to Assessment and Evaluation of Mathematical Learning

Research studies indicate new approaches and techniques in evaluating readiness to learn, interest levels in learning, and achievement of mathematical understandings. Findings indicate the values of evaluation as an integral part of the teaching-learning sequence.

Anderson, Scarvia, et. al. Priorities and directions for research and development related to measurement of young children: Report on Task 2. Princeton: Educational Testing Service, 1972.
(ERIC Document Reproduction Service No. ED 088 027).

This report is a summary of the recommendations of a panel consisting of 15 experts in child development, early childhood education and measurement which met in September, 1972, to assist the Office of Child Development in establishing priorities in improving tests and measurements for young children.

Boyer, E. Gil, Simon, Anita, and Karafin, Gail R. (Eds.). Measures of maturation: An anthology of early childhood observation instruments. (3 vols.) Humanizing Learning Program, Philadelphia: Research for Better Schools, Inc., 1973.

These three volumes describe seventy-three observation instruments for quantifying and collecting objective data concerning behavior. Volume I presents an overview of the instruments, categorizes the measurements, considers those systems which contain developmental measures, and abstracts each instrument. Volumes II and III contain the detailed anthology of the seventy-three early childhood instruments.

Circirelli, Victor G., et. al. Measures of self-concept, attitudes, and achievement motivation of primary grade children. Journal of School Psychology, 9, 1971, 383-392.

This article presents three testing instruments to measure primary grade children's self-concept, attitudinal range (attitude toward peers, home, school and society) and motivation (with respect to school learning). Results concerning the reliability and validity of each of the instruments on various populations are presented; reliabilities of these instruments seemed to be satisfactory for research use, but not for individual diagnosis. Because of their semi-protective nature, these instruments seem to trigger spontaneous responses by the child and may prove useful as a stimulus and guide for interviews rather than as tests per se.

De Vries, Rheta. The two intelligences of bright, average and retarded children. Paper presented at the meeting of the Society for Research in Child Development, Philadelphia, March 29, 1973.
(ERIC Document Reproduction Service No. ED 079 102).

This study attempted to determine if performance on Piagetian tasks can be predicted from Stanford-Benet Mental Age or IQ. Subjects were 143 children of bright, average and retarded psychometric abilities. The bright and average children were chronologically aged from 5 to 7 years; the retarded children were mentally aged from 5 to 7 years and chronologically aged from 6 to 12 years.

Fifteen Piagetian-type tasks were individually administered in three sessions. Results showed: (1) Children with higher IQ develop through Piagetian stages faster than children with lower IQ; (2) Prior to the period of concrete operations, high IQ children tend to think more in a pre-operational way than other children of the same mental age but lower IQ. This seemed to suggest that psychometric tests and Piagetian tasks reflect two kinds of intelligence; experience seemed to be an important factor in the development of Piagetian intelligence.

Johnson, Orval G., and Bommarito, James W. Tests and measurements in child development: A handbook. San Francisco: Jossey-Bass, Inc., Publishers, 1971.

This text is a guide to over 300 measures of child behavior and development not available from test publishers. These tests are listed under one of ten descriptive categories (e.g. cognition, language and number skills, motor skills). Each listing cites the author, age level, type of measure, description of measure, source of measure, reliability, validity and a bibliography of projects using the measure.

Kaufman, Alan S., and Kaufman, Nadeen L. Tests built from Piaget's and Gesell's tasks as predictors of first-grade achievement. Child Development, 43, 1972, 521-535.

This study compared the effectiveness of tests built from tasks devised by Gesell and Piaget as predictors of first-grade children attending one elementary school in Suffolk County, Long Island, New York, who were tested in kindergarten on the Gesell School Readiness Tests, a test built from Piaget's tasks and the Lorge-Thorndike Intelligence Tests, Level 1, Form A. At the end of the first grade they were administered the Stanford Achievement Test, Primary I level as a basis for comparison. The three tests were found to be excellent predictors of achievement.

Rea, Robert E., and Reys, Robert E. The Comprehensive Mathematics Inventory: An experimental instrument for assessing youngsters entering school. Journal of Educational Measurement, 7, 1970, 45-47.

The Comprehensive Mathematics Inventory (CMI) consists of 200 items arranged in seven areas: money, number, vocabulary, (Part I) geometry, measurement, recall, and pattern identification (Part II). Each item is based upon testing materials consisting of pictorial presentations and manipulative objects. In a pilot study, 727 entering first graders from six school districts and thirty classrooms in an urban area were tested by the CMI. Reliability estimates ranged from .91 to .94 for Part I and from .83 to .87 for Part II. The use of the CMI for classroom or general use is restricted by its length and the need for individual administration by trained examiners.

Schwartz, Anthony N. Assessment of mathematical concepts of five-year-old children. The Journal of Experimental Education, 37, 1969, 67-74.

This article describes the development and reliability of the first phase of the Schwartz Early Mathematics Inventory (SEMI) which was devised to assess, via a group paper and pencil instrument, the mathematical achievement of children aged 3-5. 215 kindergarten children from northern Georgia, aged from 5 years to 6 years 5 months, were administered the test in groups of twelve children. A reliability of .94 was obtained. The children had the physical and emotional attributes necessary for completing the inventory. Further research concerning validation of the instrument for three- and four-year olds is planned.

Suydam, Marilyn N. Unpublished instruments for evaluation in mathematics education: An annotated listing. Columbus: ERIC Information Analysis Center for Science, Mathematics and Environmental Education, 1974.

This text identifies tests developed and used in research publications and doctoral dissertations related to mathematics education during 1964-73. The annotated listing includes the title, developer, content, format, sample, reliability, correlations, validity and references. The supplementary listing includes tests for which no information concerning reliability or validity was available. Tests are indexed by type and author.

Part III: BIBLIOGRAPHY

The following bibliography includes research studies and books related to Mathematics and Early Childhood Education. Only materials published in the last five years are included. Some of the items on this bibliography are reviewed in the pages preceding.

AREA 1: Studies Related to the Growth and Development of Children

An annotated bibliography on children. Washington, D.C.: Bureau of Libraries and Educational Technology, 1970. (ERIC Document Reproduction Service No. ED 068 146).

Braun, Frederick G. The child from six to eight--developmental growth patterns. Paper presented at the meeting of the International Reading Association, Anaheim, California, May 6-9, 1970. (ERIC Document Reproduction Service No. ED 045 325).

Britton, James. Language and learning. Coral Gables, Florida: University of Miami Press, 1970.

Cordis, Leora L. Learning abilities of the prekindergarten child. Final report. Eugene, Oregon: University of Oregon, 1972. (ERIC Document Reproduction Service No. ED 066 225).

Dotta, Lois-Ellin. New directions for early child development programs: Some findings from research. College of Education Curriculum Laboratory. Urbana: University of Illinois, 1973. (ERIC Document Reproduction Service No. ED 081 501).

Elkind, David. Misunderstandings about how children learn. Today's Education, 61, 1972, 18-20.

Feldman, Ronald, and Coopersmith, Stanley. A resource and reference bibliography in early childhood education and developmental psychology: The affective domain. Davis, California: University of California, 1971. (ERIC Document Reproduction Service No. ED 049 817).

Fishbein, Harold D., et. al. Children's understanding of spatial relations: Coordination of perspectives. Developmental Psychology, 7, 1972, 21-33.

Gordon, Wells. Learning to code experience through language: An approach to the study of language acquisition. Revision of a paper presented at the meeting of the London School of Economics, February, 1973. (ERIC Document Reproduction Service No. ED 081 286).

Heiser, R.B. Some notes on concept formation in young children. Journal of Genetic Psychology, 114, 1969, 219-227.

Klein, Robert A. Cognitive growth in young children: Some theoretical implications pertaining to identity, language and memory. Paper presented at the meeting of the UAP-USC Conference on Piagetian Theory and the Helping professions, Los Angeles, February 16, 1973. (ERIC Document Reproduction Service No. ED 076 238).

Lavatelli, Celia, and Stendler, Faith. Readings on child behavior and development. New York: Harcourt, Brace and Jovanovich, Inc., 1972.

Levenstein, Phyllis. Cognitive growth in preschoolers through verbal interaction with mothers. American Journal of Orthopsychiatry, 40, 1970, 426-432.

Lichtenberg, Phillip, and Norton, Dolores G. Cognitive and mental development in the first five years of life: A review of recent research. Washington: U.S. Government Printing Office, 1970. (ERIC Document Reproduction Service No. ED 044 754).

McConnell, Freeman, and Horton, Kathrun B. Effects of early language training for culturally disadvantaged preschool children. Journal of School Health, 39, 1969, 661-665.

Miller, Louise B., and Dyer, Jean L. Four preschool programs: Their dimensions and effects. Louisville University: Department of Psychology, 1972. (ERIC Document Reproduction Service No. ED 069 411).

Moore, Ruth C., and Ogletree, Earl J. A comparison of the readiness and intelligence of first grade children with and without a full year of Head Start training. Education, 93, 1973, 266-270.

Murphy, Lois Barclay. Foundations for good beginnings. Young Children, 25, 1969, 5-11.

Reese, Clyde, and Morrow, Robert. Socioeconomic mix: Effects on disadvantaged children in preschool child development programs. Conway, Ark.: St. College of Ark., 1971. (ERIC Document Reproduction Service No. ED 067 147).

Schulman, Janice B., and Prall, Robert C. Normal child development: An annotated bibliography of articles and books published, 1950-1969. New York: Grune & Stratton, Inc., 1971.

Shantz, Carolyn U. Egocentrism in children: Its generality and correlates. Head Start evaluation and research center. Detroit: Merriff Palmer Institute, 1969. (ERIC Document Reproduction Service No. ED 038 687).

Strom, Robert D. Toy talk: A medium for respect. Theory into Practice, 8, 1969, 186-191.

Witz, E.G., and Easley, J.A., Jr. Analysis of cognitive behavior in children. Final report. University of Illinois Curriculum Laboratory. Urbana: University of Illinois Press, 1972. (ERIC Document Reproduction Service No. ED 976 261).

AREA 2: Studies Related to the Development of Mathematical Thought in Children

Baker, Nancy E., and Sullivan, Edmund V. The influence of some task variables and socioeconomic class on the manifestation of conservation of number. Journal of Genetic Psychology, 114, 1970, 21-30.

Brainerd, Charles J., and Brainerd, Susan H. Order of acquisition of number and quantity conservation. Child Development, 43, 1972, 1401-1406.

Brearley, Molly. The teaching of young children: Some applications of Piaget's learning theory. New York: Schoken Books, 1970.

Brown, Ann L. Conservation of number and continuous quantity in normal, bright, and retarded children. Child Development, 44, 1973, 376-379.

Bucher, Bradley, and Scheider, Robert E. Acquisition and generalization of conservation by pre-schoolers using operant training. Journal of Experimental Child Psychology, 16, 1973, 187-204.

Carpenter, Thomas Phelps. The role of equivalence and order relations in the development and coordination of the concepts of unit size and number of units in selected conservation type measurement problems. (Unpublished doctoral dissertation, University of Wisconsin, 1971). Dissertation Abstracts International, 32, 1972, 448A.

Craig, Grace J., Love, Jean A. and Olim, Ellis G. Perceptual judgments in Piaget's conservation-of-liquid problem. Child Development, 44, 1973, 372-375.

Denney, Nancy Wadsworth. A developmental study of free classification in children. Child Development, 43, 1972,221-232.

Divers, Benjamin P., Jr., and Steffe, Leslie P. The ability of kindergarten and first grade children to use the transitive property of three length relations in three perceptual situations. Paper presented at the meeting of the American Educational Research Association, New York, February 4, 1971. (ERIC Document Reproduction Service No. ED 048 936).

DeVries, Rheta. Relationships among Piagetian achievement and intelligence assessments. Paper presented at the meeting of the American Educational Research Association, New Orleans, February, 1973. (ERIC Document Reproduction Service No. ED 079 101).

Friedman, Myles, et. al. The development of problem-solving skills in early childhood. Committee on Educational Research. Columbia: University of South Carolina, 1971. (ERIC Document Reproduction Service No. ED 079 101).

Gelman, Rochel. Logical capacity of very young children: Number invariance rules. Child Development, 43, 1972, 75-90.

Goldschmid, Marcel I. Different types of conservation and nonconservation and their relation to age, sex, IQ, MA and vocabulary. Child Development, 38, 1967, 1229-12243.

Gotts, Edward Earl. Some determinants of young children's attribute attending. Merrill Palmer Quarterly, 19, 1973, 261-273.

Haney, Joanne H., and Hooper, Frank H. A developmental comparison of social class and verbal ability influences on Piagetian tasks. Journal of Genetic Psychology, 122, 1973, 235-245.

Harper, E. Harold, Steffe, Leslie P., and Van Engen, Henry. An evaluation of teaching conversation of numerousness. School Science and Mathematics, 69, 1969, 287-296.

Koshinsky, Claudia, and Hall, Alfred E. The developmental relationship between identity and equivalence conservation. Journal of Experimental Child Psychology, 15, 1973, 419-424.

Little, Audrey. A longitudinal study of cognitive development in young children. Child Development, 43, 1972, 1024-1036.

Lovell, Kenneth R. Intellectual growth and understanding mathematics Implications for teaching. Arithmetic Teacher, 19, 1972, 277-282.

Lumsden, Ernest A., and Kling, J. Kenneth. The relevance of an adequate concept of "bigger" for investigations of size conservation: A methodological critique. Journal of Experimental Child Psychology, 8, 1969, 82-91.

Mermelstein, Egon, and Meyer, Edwina. Conservation training techniques and their effects on different populations. Child Development, 40, 1969, 471-490.

Miller, Patricia H., Grabowski, Teddy L., and Heldmyer, Karen H. The role of stimulus dimensions in the conservation of substance. Child Development, 44, 1973, 646-650.

Miller, Scott A., and Lipps, Leann. Extinction of conservation and transitivity of weight. Journal of Experimental Child Psychology, 16, 1973, 388-402.

Murray, Frank B. Stimulus mode and the conservation of weight and number. Journal of Educational Psychology, 61, 1970, 287-291.

Overbeck, Carla, and Schwartz, Marian. Training in conservation of weight. Journal of Experimental Child Psychology, 9, 1970, 253-264.

Owens, Douglas T., and Steffe, Leslie P. Differential performance of kindergarten children on transitivity of three matching relations. Paper presented at the meeting of the American Educational Research Association, New York, February 4, 1971. (ERIC Document Reproduction Service No. ED 048 942).

Palfrey, C.F. Piaget's questions to young children: An analysis of their structure and content. Educational Review, 24, 1972, 122-31.

Papalia, Diane E., and Hooper, Frank H. A developmental comparison of identity and equivalence conservations. Journal of Experimental Child Psychology, 12, 1971, 347-361.

Peisach, Estelle, and Wein, Norman. Relationship of conservation explanations to item difficulty. Journal of Genetic Psychology, 117, 1970, 167-180.

Resnick, Lauren B. (ed.) Hierarchies in children's learning: A symposium. Instructional Science, 2, 1973, 331-372.

Rohr, Judith Ann Green. The relationship of the ability to conserve on Piagetian tasks to achievement in mathematics. (Unpublished doctoral dissertation, The University of Tennessee, 1973). Dissertation Abstracts International, 34, 1973, 2398A.

Roll, Samuel. Reversibility training and stimulus desirability as factors in conservation of number. Child Development, 41, 1970, 501-507.

Rose, Susan Ann. Acquiescence and conservation. Child Development, 44, 1973, 811-814.

Rose, Susan A., and Blank, Marion. The potency of context in children's cognition: An illustration through conservation. Child Development, 45, 1974, 499-502.

Schnall, Melvyn, Alter, Ellen, Swanlund, Terry and Schweitzer, Thomas. A sensory-motor context affecting performance in a conservation task: A closer analogue of reversibility than empirical return. Child Development, 43, 1972, 1012-1023.

Van Wagenen, R. Keith, and Walker, Alice A. Formulation and quick integration of mathematics concepts in the child during the first school year. Tempe, Arizona: Arizona State University, 1973. (ERIC Document Reproduction Service No. ED 085 116).

Wallach, Lise, Wall, A. Jack, and Anderson, Lorna. Number conservation: The roles of reversibility, addition-subtraction, and misleading perceptual cues. Child Development, 38, 1967, 425-442.

Wang, Margaret C., et. al. The sequence of development of some early mathematics behaviors. Pittsburg University: Learning Research and Development Center, 1970. (ERIC Document Reproduction Service No. ED 052 947).

Weaver, J. Fred. Some concerns about the application of Piaget's theory and research to mathematical learning and instruction. Arithmetic Teacher, 19, 1972, 263-270.

AREA 3: Studies Related to the Identification of Appropriate Mathematical Content for Young Children

Duckworth, Eleanor. The having of wonderful ideas. Harvard Educational Review, 42, 1972, 217-231.

Elardo, Phyllis T., and Caldwell, Bettye M. The Kramer adventure: A school for the future? Childhood Education, 50, 1974, 143-152.

Friedman, Ruth. Characteristics and early education needs of disadvantaged children. Child Welfare, 53, 1974, 93-97.

Harvey, John G. The content of arithmetic included in a modern elementary mathematics project. Research and Development Center for Cognitive Learning. Madison: University of Wisconsin, 1971. (ERIC Document Reproduction Service No. ED 070 656).

Heard, Ida May. Developing geometric concepts in the kindergarten. The Arithmetic Teacher, 16, 1969, 229-230.

Hodgen, Laurel, et. al. School before six: A diagnostic approach. (2 vols.) St. Louis: The CEMREL Institute, 1974.

Kilpatrick, Jeremy. Problem solving in mathematics. Review of Educational Research, 39, 1969, 523-534.

Lazarus, JoAnn Mannise. The development of a kindergarten mathematics program: Implemented through inservice teacher education. (Unpublished doctoral dissertation, Wayne State University, 1972). Dissertation Abstracts International, 33, 1972, 2220A-2221A.

Pollio, Howard R., and Whitacre, Janet D. Some observations on the use of natural numbers by preschool children. Perceptual and Motor Skills, 30, 1970, 167-174.

Scott, Ralph. Research and early childhood: The Home Start Project. Child Welfare, 53, 1974, 112-119.

Shah, Sair Ali. Selected geometric concepts taught to children ages seven to eleven. The Arithmetic Teacher, 16, 1969, 119.

Shah, Sair Ali. Selected topological concepts taught to children ages six to nine. A report read at the Annual Meeting of the AERA, 1971.

Steffe, Leslie P. and Carey, Russell L. Equivalence and order relations as interrelated by four and five year old children. Journal of Research in Mathematics Education, 3, 1972, 77-88.

Thyer, Dennis, and Maggs, John. Teaching mathematics to young children. New York: Holt, Rinehart, and Winston, 1971.

Weaver, J. Fred. The symmetric property of the equality relation and young children's ability to solve open addition and subtraction sentences. Journal for Research in Mathematics Education, 4, 1973, 45-56.

Williford, Harold J. A study of transformational geometry instruction in the primary grades. Journal for Research in Mathematics Education, 3, 1972, 260-271.

AREA 4: Studies Related to Strategies of Teaching Mathematical Concepts to Young Children

Anastasiow, Nicholas J., et. al. A comparison of guided discovery, discovery and didactic teaching of math to kindergarten children. American Educational Research Journal, 7, 1970, 493-509.

Engel, Carol D., and Lerch, Harold H. A comparison of first grade children's abilities on two types of arithmetical practice exercises. School Science and Mathematics, 71, 1971, 327-334.

Fennema, Elizabeth H. The relative effectiveness of a symbolic and concrete model in learning a mathematical principle. Journal For Research in Mathematics Education, 3, 1972, 233-238.

Keane, Dorothy Louise. An exploration of the Nuffield Mathematics Teaching Project with reference to its current use in selected British schools. (Unpublished doctoral dissertation, Wayne State University, 1973). Dissertation Abstracts International, 34, 1973, 3029A.

Peters, Donald L. Discovery learning in kindergarten mathematics. Journal for Research in Mathematics Education, 1, 1970, 76-87.

Romberg, Thomas A., and Gilbert, Lynn E. The effect of training on length of performance of kindergarten children on nonstandard related tasks. Journal of Research in Mathematics Education, 3, 1972, 69-76.

Stafford, Don G., and Renner, John W. SCIS helps the first grader to use logic in problem solving. School Science and Mathematics, 71, 1971, 159-164.

Trueblood, Cecil R. A model for using diagnosis in individualizing mathematics instruction in the elementary school classroom. Arithmetic Teacher, 18, 1971, 505-511.

Weber, Audra Wheatly. Introducing mathematics to first grade children: Manipulative vs. paper and pencil. (Unpublished doctoral dissertation, University of California, Berkeley, 1969). Dissertation Abstracts International, 30, 1970, 3372A-3373A.

AREA 5: Studies Related to the Assessment and Evaluation of Learning

Adkins, Dorothy C. Objective measurement of emerging affective traits in preschool children. Paper presented at the meeting of the American Psychological Association, Honolulu, September 6, 1972. (ERIC Document Reproduction Service No. ED 069 675).

Anderson, Scarvia, et. al. Priorities and directions for research and development related to measurement of young children. Report Task 2. Princeton, N.J.: Educational Testing Service, 1972. (ERIC Document Reproduction Service No. ED 088 027).

Boyer, E. Gil, Simon, Anita, and Karafin, Gail R. (Eds.) Measures of maturation: An anthology of early childhood observation instruments. (3 vols.) Humanizing Learning Program. Philadelphia: Research for Better Schools, Inc., 1973.

Chittenden, Edward A., and Bussis, Anne M. Open education: Research and assessment strategies. Paper presented at the meeting of the National Association for the Education of Young Children, Minneapolis, November 6, 1971. (ERIC Document Reproduction Service No. ED 060 932).

Circirelli, Victor G., et. al. Measures of self-concept, attitudes, and achievement motivation of primary grade children. Journal of School Psychology, 9, 1971, 383-392.

Circus: Comprehensive assessment in nursery school and kindergarten. Proceeding of a symposium presented at the meeting of the American Psychological Association, Montreal, August 21, 1973. (ERIC Document Reproduction Service No. ED 087 786).

Devries, Rheta. The two intelligences of bright, average, and retarded children. Paper presented at the meeting of the Society for Research in Child Development, Philadelphia, March 29, 1973. (ERIC Document Reproduction Service No. ED 079 102).

Evaluating children's progress: A rating scale for children in day care. Atlanta: Southern Regional Education Board, 1973. (ERIC Document Reproduction Service No. ED 078 949).

Johnson, Orval G., and Bommarito, James W. Tests and measurements in child development: A handbook. San Francisco: Jossey-Bass, Inc., Publishers, 1971.

Kaufman, Alan S., and Kaufman, Nadeen L. Tests built from Piaget's and Gesell's tasks as predictors of first grade achievement. Child Development, 43, 1972, 521-535.

Rea, Robert E., and Reys, Robert E. The Comprehensive Mathematics Inventory: An experimental instrument for assessing youngsters entering school. Journal of Educational Measurement, 7, 1970, 45-47.

Schwartz, Anthony N. Assessment of mathematical concepts of five-year-old children. The Journal of Experimental Education, 37, 1969, 67-74.

Suydam, Marilyn N. Evaluation in the mathematics classroom. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics and Environmental Education, 1974.

Suydam, Marilyn N. Unpublished instruments for evaluation in mathematics education: An annotated listing. Columbus: ERIC Information Analysis Center for Science, Mathematics and Environmental Education, 1974.

Williams, Robert. Testing for number readiness: Application of the Piagetian theory of the child's development of the concept of number. The Journal of Educational Research, 64, 1971, 394-396.